Complex Surface Fabrication via Developable Surface Approximation: A Survey

Chao Yuan, Nan Cao, and Yang Shi

Abstract—Complex surfaces are commonly observed in various applications and have significant value in enhancing comfort, aesthetics, and functionality. However, their fabrication often involves complex and costly processes. To simplify the fabrication difficulty, significant research has focused on using 3D developable surfaces to approximate target 3D surfaces. This process involves converting target 3D surfaces into developable surfaces and then flattening them into 2D patterns. Since the geometric and topological diversity of target surfaces, this task is both comprehensive and intricate, encompassing multiple aspects from design to fabrication. In this paper, we review relevant technologies and methods in fabrication processes, classify them, and summarize a pipeline from design to fabrication. This provides a comprehensive introduction to the field for researchers and practitioners. Through the analysis of relevant literature, we also discuss some of the research challenges and future research opportunities.

Index Terms—Developable Surfaces, Developable Approximation, Digital Fabrication, Physicalization.

1 Introduction

THREE-DIMENSIONAL thin-shell surfaces are complex surfaces with non-zero Gaussian curvature, widely used in architecture, product design, sculpture, mechanical materials, and data physicalization, especially in wearable products that conform to human curvature. They hold significant value in enhancing comfort, aesthetics, and functionality. However, 3D surfaces involve more complex and costly manufacturing processes compared to planar or single-curved surfaces. Conventional fabrication techniques, such as CNC milling and molding, result in high costs, while general 3D printing requires extensive support structures and complex post-processing. To simplify the fabrication of 3D surfaces, existing research focuses on the complex surface fabrication via developable surface approximation. This process involves converting 3D surfaces into planar patterns. The fabrication steps include: (1) first converting the target 3D surface into 3D developable surfaces or multiple planar pieces with minimal distortion, (2) then flattening them into one or more 2D patterns, and (3) finally fabricating and assembling 2D patterns using appropriate approaches. This fabrication process offers many advantages, such as reducing manufacturing complexity, conserving materials, and lowering transportation costs. In addition, developable surfaces are closely related to flexible manufacturing [1], which can improve the efficiency and intelligence of manufacturing.

Developable surfaces can be formally defined as surfaces with zero Gaussian curvature. According to this definition, at least one principal curvature is zero, which includes cylindrical surfaces, conical surfaces, planes, and tangent developable surfaces. These surfaces can be flattened into a plane through coordinate transformations without shape distortion. In related applications, planes can be folded into intricate 3D structures, inspiring innovations such as folding architecture and solar panels on satellites. Conversely, transforming 3D surfaces into developable surfaces serves as an effective manufacturing applicable across various fields. This paper emphasizes complex surface fabrication processes that transition from 3D to 2D and back to 3D.

However, the fabrication process is a complex task with various challenges. The diversity in design requirements leads to a wide range of geometric and topological variations in the target shapes, thereby increasing the complexity of the task. The converting process from target surfaces to developable surfaces usually involves shape cutting, which significantly impact subsequent fabrication and assembly complexities. During the process of the shape developable optimization, deformation errors can occur due to factors such as the shape's topological structure and the algorithms employed. Additionally, 2D patterns generated by different flattening operations are further constrained by fabrication and assembly considerations. These examples highlight some of the challenges inherent in the fabrication process.

These difficulties and diverse methodological challenges have motivated extensive investigation into addressing various aspects of fabricating complex surfaces. For instance, in the fabrication of CAD models, some early research explored methods for piecewise developable approximation, such as the "geodesic curvature preservation" [2], strips approximation based on mesh simplification [3] and the 3D shapes developable approximation by modeling cylinders [4]. As research progressed, more developable approximation methods are proposed to meet the requirements of more precise approximation, fewer pieces, and other specific design needs. These methods not only simplify fabrication processes and reduce costs but also inspire new design directions and styles.

The purpose of this survey is to provide researchers and practitioners with a comprehensive overview of complex surface fabrication via developable surface approximation.

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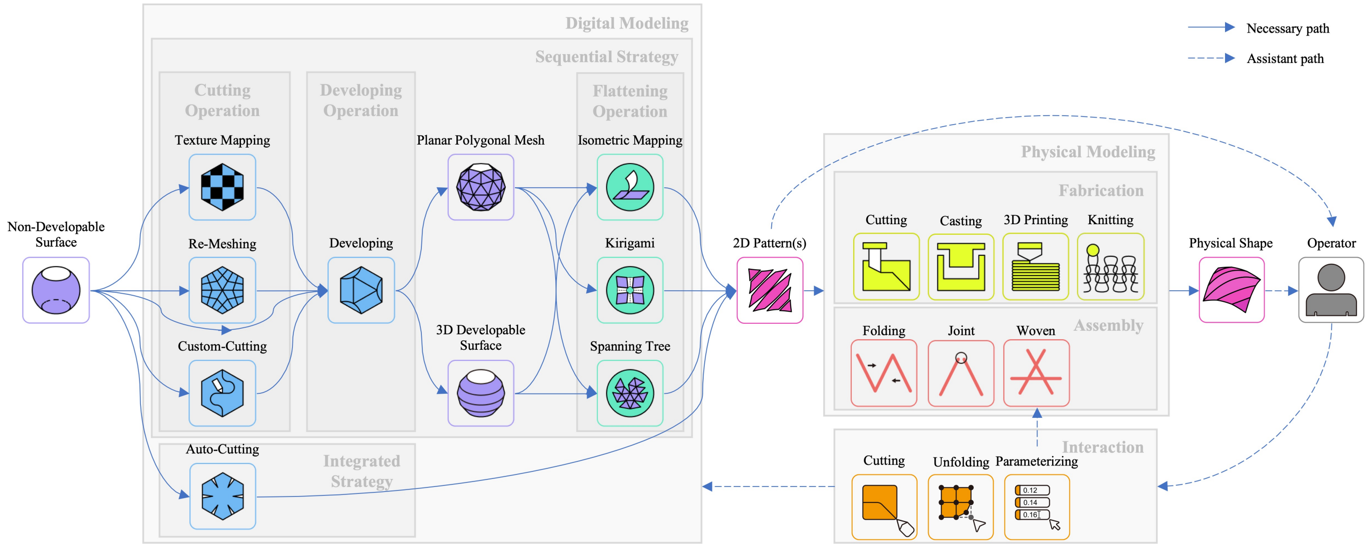


Fig. 1. Pipeline of complex surface fabrication processes by using developable surface approximation from digital modeling to physical modeling. Digital modeling: the process takes a target shape as input and produces one or more 2D patterns for fabrication. If the sequential strategy is employed, the target shape needs to be converted into a planar polygonal mesh or a shape consisting of one or more 3D developable surfaces, which is then flattened into one or more 2D patterns. Physical modeling: the process takes 2D patterns as input and outputs the actual produced shapes. This process involves fabrication and assembly methods. Interactive assistance: alternative operations for better or more convenient optimizing specific processes. Note that in digital modeling, certain operations may yield different outcomes, leading to distinct subsequent steps. For example, if re-meshing produces a polygonal mesh, it can be optimized into a planar polygonal mesh before flattening. However, if the output is a series of strips, they must first be converted into a shape consisting of developable surfaces before flattening.

Through literature review and analysis, we identify relevant fabrication processes and operations, summarize a pipeline (see Fig. 1), and analyze various techniques from design to fabrication. Based on different design requirements, we use this pipeline to organize the corresponding operations and determine some general fabrication processes. Additionally, we explore research directions and applications that have received less attention in previous work and present challenges for future research.

2 Related Surveys and Methodology

In this section, we first review surveys relevant to fields of developable surfaces. After that, we introduce our method for collecting and categorizing papers.

2.1 Related Surveys

This section collects the recent surveys on developable surfaces. These surveys are classified into two categories. One focuses on general methods and techniques of developable surfaces, and the other focuses on applications based on folding forms.

In the first type, Bhanage [5] reviewed developable surfaces in the field of sheet metal. Nejur and Steinfeld [6] reviewed some mesh cutting algorithms by mesh dual graph applied in "generative architectural design". Zhang and Zheng [7] reviewed and classified developable surface techniques in geometric modeling. Compared to these surveys, the main features and contributions of our work include reviewing comprehensive fabrication processes involving technologies and methods from design to fabrication, summarizing a general pipeline consisting of multiple fabrication processes, and providing potential opportunities for integrating developable surfaces with other methods.

In the second type, origami structure, a special developable surface, can be used in a variety of fields. In mechanical engineering, origami structures offer unique material properties. Peraza-Hernandez et al. [8] reviewed "active materials" based on origami. Turner et al. [9] reviewed mechanical devices based on origami applications. Johnson et al. [10] proposed an overview of applications in biomedical devices by origami structures. Li et al. [11] focused on the research of geometry and properties of origami materials. Shah et al. [12] reviewed deployable antennas based on origami methods. Meloni et al. [13] presented origami methods for engineering applications from 2015 to 2020. Fonseca et al. [14] proposed an overview of "origami-inspired systems and structures" for smart materials. Some reviews focus on architectural applications. Doroftei et al. [15] focused on the overview of applications of "foldable plate structures" in architecture. In product design, Meloni et al. [13] reviewed product designs based on origami structures. In the field of origami art and design, Demaine et al. [16] reviewed curved folding in art and design.

2.2 Survey Methodology and Taxonomy

To provide a comprehensive review of existing studies, we collected relevant papers from computer graphics journals and conferences by using two main approaches: search-driven and citation-driven selection. For the former, we first used a keyword approach in the ACM literature search database to obtain the initial papers based on the extended schema of the ACM Guide to Computing Literature. The relevant keywords have multiple expressions, so we listed as many expressions as possible and connected these words by logical "OR" to obtain a total of 624 initial papers. We then reviewed the abstracts of each paper and then filtered out the papers that did not meet the requirements. The reason is that these papers only mention the keywords of

”developable surfaces” but not the study of developable surfaces. For the citation-driven selection, we extended the collection by using publications that we knew in advance about the topic and relevant citations from the articles collected based on the search drive. The following list shows these keywords and their alternative expressions:

- **Developable surfaces**: ”developable surface”, ”developable surfaces”, ”developable structure”, ”developable structures”, ”developable approximation”, ”developable approximations”, developability, origami and kirigami.
- **Mesh segmentation**: ”mesh segmentation”, ”mesh separation”, ”meshes segmentation”, ”meshes separation”.

We eventually collected a total of 150 articles (collected to 2023), including 75 papers on the digital modeling process. and concentrate on *ACM TOG* (42 papers), *Computer-Aided Design* (10 papers), *Computers & Graphics* (4 papers), and *Computer Graphics Forum* (4 papers).

Based on our analysis of collected literature, We have summarized a comprehensive pipeline involving multiple fabrication processes from design to fabrication. This pipeline consists of two stages: digital modeling and physical modeling. Since the geometric and topological diversity of shapes, different methods are used. The digital modeling stage aims to transform 3D shapes into 2D patterns for fabrication. In the stage, some methods can directly generate 2D patterns, but there are also some methods that generate 3D developable surfaces or polygonal meshes, which need to be further mapped onto a 2D plane. Additionally, some interactive assistance is essential to help operators detailedly fine-tune shape during cutting or flattening, based on feedback from the visual results. In the physical modeling stage, we divide it into two key processes: fabrication and assembly. This stage aims to produce physical shapes from digital models and finalize the result through assembly. According to the physical modeling result, The operator can also adjust the parameters of the CNC equipment to precisely production.

The rest of the contents are organized as follows. We analyzed the corresponding digital modeling methods in section 3. Section 4 analyzes the physical modeling methods. Then, Section 5 summarizes the interactive assistance. Section 6 summarizes some of the application areas based on these fabrication processes by using developable surface approximation. Finally, we discuss challenges and opportunities in Section 7 and conclude in Section 8.

3 Digital Modeling

In this section, we provide categories of related techniques in digital modeling process. The process begins with a target 3D surface as input. Our objective is to compute 2D patterns of the target surface under controlled distortion conditions, such that the fabricated 2D patterns can approximate the target surface through appropriate assembly methods.

Since the diversity of geometries and design requirements, the digital modeling process often involves many methods including cutting, developing and flattening. For cutting operations, since the non-developable surfaces exist distortion when directly flattening them, cutting operations are required to minimize the shape distortion. Additionally, cutting operations often need to meet requirements for simplifying fabrication difficulty. Therefore, either the number of pieces is as small as possible, or the pieces as regular as possible to facilitate fabrication or assembly. For developing operations, the aim is to convert the target surface into a shape consisting of one or more 3D developable surfaces, or a shape consisting of multiple planar pieces, where the shape needs to approximate the target surface as closely as possible. For flattening operations, the aim is to convert the shape into one or more 2D patterns.

In the digital modeling process, depending on the specific task requirements, two strategies are employed. One is the sequential strategy which involves performing cutting, developing and flattening operations sequentially, while the other is the integrated strategy which directly converting the target surface into 2D patterns. Therefore, for the sequential strategy, **meshing** and **custom-cutting** are employed to generate cuts onto the target surface, **developing** is employed to form 3D developable surfaces or a planar polygonal mesh to approximate the target surface, and **flattening** is employed to form 2D patterns for fabrication. Additionally, for the integrated strategy, **auto-cutting** directly forms 2D patterns with cuts. Table 1 shows the classification of strategies and methods in digital modeling process, and Figure 2 shows four examples of the basic process of the sequential strategy.

3.1 Meshing

Converting a target surface into numerous pieces, which is a common idea in architecture for building complex surfaces. To simplify fabrication, these pieces are as regular and similar as possible. The conversion process is called meshing, and these decomposited pieces are often triangles, quadrangles, hexagons, or strips. The combination of numerous pieces allows meshing to approximate the target surface more closely, and each piece exhibits a small curvature, facilitating easier flattening. Additionally, if these pieces are similar, fabrication can be accelerated and costs reduced by reducing the number of piece types through clustering [92]–[94]. Therefore, meshing, as a cutting operation of the sequential strategy, often involves determining how to generate cuts to meet specific requirements. Based on the reviewed literature, we summarize two meshing methods: **texture mapping** for generating cut patterns in a 2D parametric domain, and **re-meshing** for generating cuts based on geometric properties.

Texture Mapping. One approach to fabricating the target surface using numerous similar pieces is texture mapping. The core concept involves utilizing surface parameterization to map a grid as textures onto the target surface. The process consists of two main steps: (1) Surface parameterization, which establishes a mapping relationship between the target shape and a 2D parameter domain; (2) Creating a regular grid as textures on the parameter domain, mapped onto the target surface to form numerous pieces. Subsequently, these pieces need to be converted into planar pieces to form a planar polygonal mesh. The process is as illustrated in Figure 2a, where surface parameterization as the core method plays a crucial role which facilitates the texture creation and mapping. For instance, Eck et al. [17] computed a Voronoi diagram based on the 2D parametric domain of the target

TABLE 1
Classification of strategies and methods in the digital modeling process

Operation Types	Methods		Ability	Examples
Cutting Operations	Meshing	Texture Mapping	Generating regular polygonal meshes	[17]–[20]
		Re-Meshing	Generating strips or polygonal meshes	[17], [21]–[49]
	Custom-Cutting		Forming piecewise surfaces	[41], [50]–[63]
Developing Operation	Developing		Generating 3D developable surfaces or planar polygonal meshes	[26], [48], [50], [52]–[55], [59], [64]–[76]
Flattening Operations	Isometric Mapping		Forming 2D patterns	[77]–[83]
	Kirigami		Forming a 2D piece	[63], [75], [84], [85]
	Spanning Tree		Forming a small number of 2D pieces	[85], [86]
Integrated Strategy	Auto-Cutting		Forming 2D patterns	[42], [87]–[91]

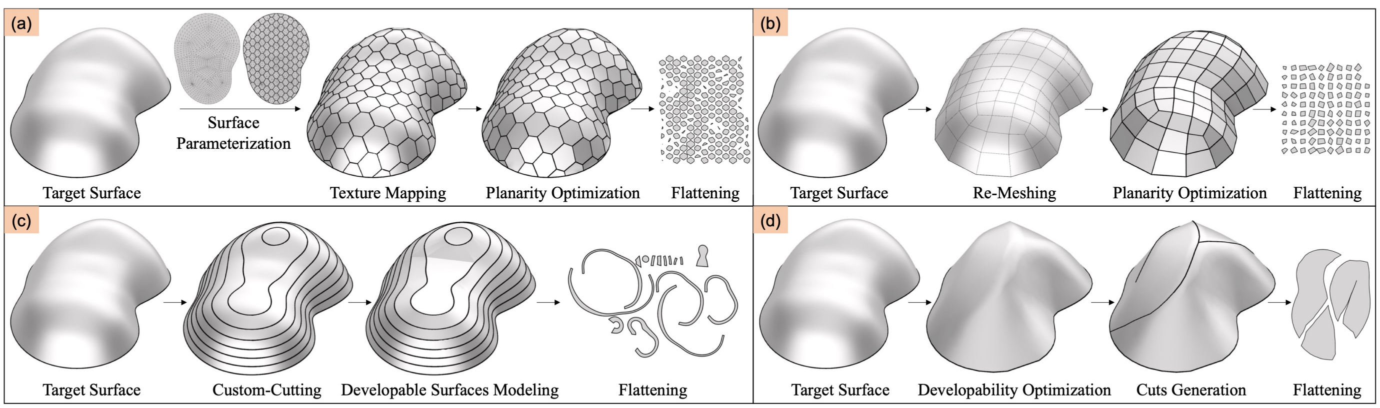


Fig. 2. Four examples of the basic process of sequential strategy: (a) Process from meshing using texture mapping to planarity optimization and flattening; (b) Process from meshing using re-meshing to planarity optimization and flattening; (c) Process from custom-cutting to developable surfaces modeling and flattening; (4) Process from developability optimization to flattening.

surface, followed by meshing the surface. Zheleznyakova et al. [18] applied texture mapping to construct a uniform triangular mesh. First, interacting nodes are distributed at optimal locations in the parametric domain of the NURBS surface using molecular dynamics simulation. Then, well-formed triangles are generated by connecting these nodes through Delaunay triangulation. Finally, the nodes are mapped from the parametric domain to the NURBS surface. Gao et al. [19] proposed a grid generation method where the grid is generated in a parametric domain of the target surface by guide line method. Peng et al. [20] proposed a framework to generate mesh patterns that consist of a hybrid of both triangles and quads, which are then mapped onto the target surface based on its parametric domain.

Re-Meshing. Another approach to fabricating the target surface using numerous similar pieces is re-meshing. In this process, the pieces are formed directly on the target surface without using the 2D parametric domain. The key step of the process is how to reconstruct a mesh or grid and then form numerous pieces. Subsequently, if these pieces are non-planar polygons, they need to be converted into planar pieces to form a planar polygonal mesh, and the process is illustrated in Figure 2b. If pieces are strips, two trace curves of the strip can be used to model a developable surface (in Section 3.2) instead of planarity optimization to form a planar polygonal mesh. According to collected studies, existing research implements re-meshing by methods including: *geodesic*, *principal curvature*, *vector field*, and *tessellation*.

Geodesic. A geodesic is the shortest path between two points on a local surface, with its tangential vector always lying on the surface. Using the geodesic method, a curve can be constructed directly on the surface. Therefore, a regular net can be generated on the surface by applying suitable rules. Constant-width nets are created by using equidistant geodesic curves to divide the target shape into uniform strips. If the width of these strips significantly deviates from a constant value, breakpoints are introduced. For example, Pottmann et al. [21] demonstrated the creation of divided nets using constant-width nets on the surface (see Fig. 3a), while Jiang et al. [22] introduced the concept of ”pseudo-geodesics” (non-strict geodesic curves) to keep the width of the strips as constant as possible. Orthogonal nets consist of two families of curves that are locally perpendicular

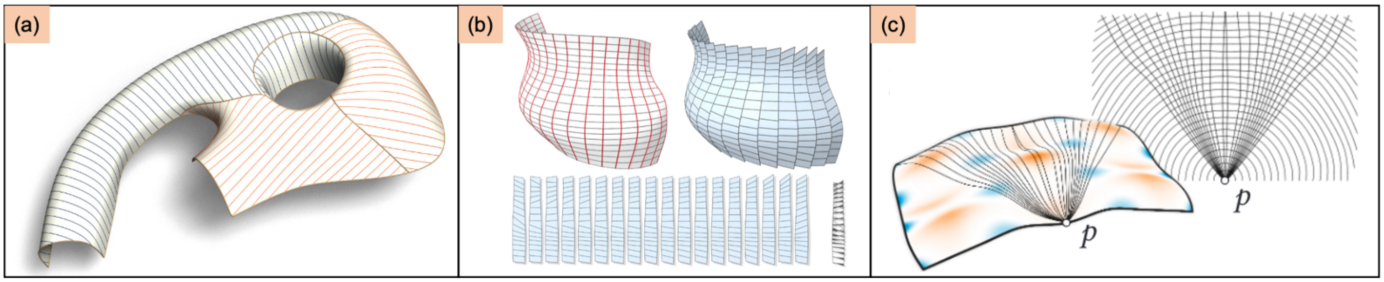


Fig. 3. Meshing of the target shapes based on geodesic: (a) Constant width geodesic curves on surfaces [21]. (b) Surfaces split by orthogonal nets [27]. (c) Constructing shortest geodesic by distance fields [28].

when passing through a point. Rabinovich et al. [23] proposed ”discrete orthogonal geodesic nets” for consistent deformation of developable surfaces. Subsequently, they developed methods for deforming discrete surfaces with curve constraints [24] and interactive design [25]. Ion et al. [26] approximated curved geometries with piecewise developable surfaces based on these nets. Wang et al. [27] introduced ”discrete geodesic parallel coordinates,” where one family is geodesic, and the other is locally orthogonal to it (see Fig. 3b). The distance field is a meshing operation involving the equidistant offsetting of curves around a point on the target surface, thereby creating a geodesic net. This technique is applied in various contexts, such as creating deployable grids [28], improving deployable grids with non-convex hull structures [29], and generating knitting paths on surfaces [30]. The distance field approach facilitates the direct production of non-developable surfaces without the need for cutting and stitching operations (see Fig. 3c) and contributes to advancements in 3D knitting path generation (see Fig. 8d).

Principal Curvature. Curvature is a measure of the bending condition in a curve, and it is extended to surfaces. Let $df(X)$ be a unit tangent direction at a point X on the surface, X has a normal vector N, and consider a plane containing both $df(X)$ and N that intersects the surface in a curve, whose curvature at the X is called the normal curvature. There are many normal curvatures at the X on the surface. Principal curvature refers to the two maximum and minimum normal curvatures values. These values are associated with two orthogonal directions known as principal directions, which are perpendicular to each other. Several studies generate orthogonal nets on curved surfaces based on the principal curvature. Joo et al. [31] proposed an algorithm for calculating differential geometric properties of curvature lines of parametric surfaces. A 3D shape is segmented into strips using orthogonal curvature lines [32] (see Fig. 4a). Principal curvature is used to re-mesh 3D shapes into quad meshes [33], and shapes are re-meshed into quad meshes using vector fields constructed based on the direction of principal curvature [34].

Vector Field. A vector field can be represented by vectors at each point on a 2D manifold, where each position has a specific direction and magnitude. Vector fields are often generated based on geometric properties such as principal curvature and geodesics. Vaxman et al. [35] provided a review of different directional fields. Vector fields are frequently used to reconstruct quad meshes, typically involving optimizations related to length and orientation preserving. Regarding re-meshing by a vector field, Sageman-Furnas et al. [36] utilized a vector field to create a global discrete ”Chebyshev network”, where all edge lengths of the quad mesh are equal, to approximate a 3D shape. Jiang et al. [37] employ a killing vector field in a given surface (where points on the given surface move the same distance along the killing vector field) to construct a quad mesh with a checkerboard pattern. These quad mesh faces can be easily manufactured by mapping them onto a regular surface, such as a rotational surface(see Fig. 4b). He et al. [38] constructed a tangent vector field based on minimum principal curvatures to generate flank milling tool paths with quasi-developable surfaces. Verhoeven et al. [34] constructed a vector field by considering the direction of principal curvature, aligning the vector field with the re-meshed meshes. Cross-field refers to a vector field with 4-rotational symmetry, and it is also used for meshing. For instance, masonry in a 3D shell is implemented by reconstructing a quad network [39]. The cross-field re-meshing method is employed to align sharp characteristics of a given 3D shape [40]. Additionally, an interactive 3D garment segmentation method is proposed based on a cross-field, and users split a garment by defining start points [41] (see Fig. 9a).

Tessellation. Tessellation involves the use of polygons to form the target shape. Often, tessellation is driven by aesthetic considerations, and polygons can be obtained by optimizing the dual graph of a triangular mesh. When unit polygons are convex, the dual graph of a triangular mesh serves to construct a polygonal mesh [42]. However, non-uniform triangular meshes lead to non-uniform polygonal meshes. One approach to address this is the construction of a ”circle packing (CP)” mesh, where the incircles of two triangles sharing a common edge have the same contact point [43], [44]. This method can generate a uniform polygonal mesh through the Voronoi diagram. Alternatively, a polygonal mesh can be generated by constructing a Voronoi diagram on the target shape using seed points [17]. Additionally, existing research employs physics-based optimization methods to obtain an uniform polygonal mesh, i,e., nodes are randomly generated on the target surface, followed by uniform distribution using defined ”repulsion” forces. [45]–[47]. When unit polygons are concave, existing studies construct concave textures via geometric rules [42], [48]. For irregular concave polygons, Chen et al. utilized an ”attraction-repulsion” optimization method to tessellate irregular polygons on the target surface [49] (see Fig. 4c).

3.2 Custom-Cutting

To more freely meet specific design requirements, custom cutting is essential. This method also serves as cutting

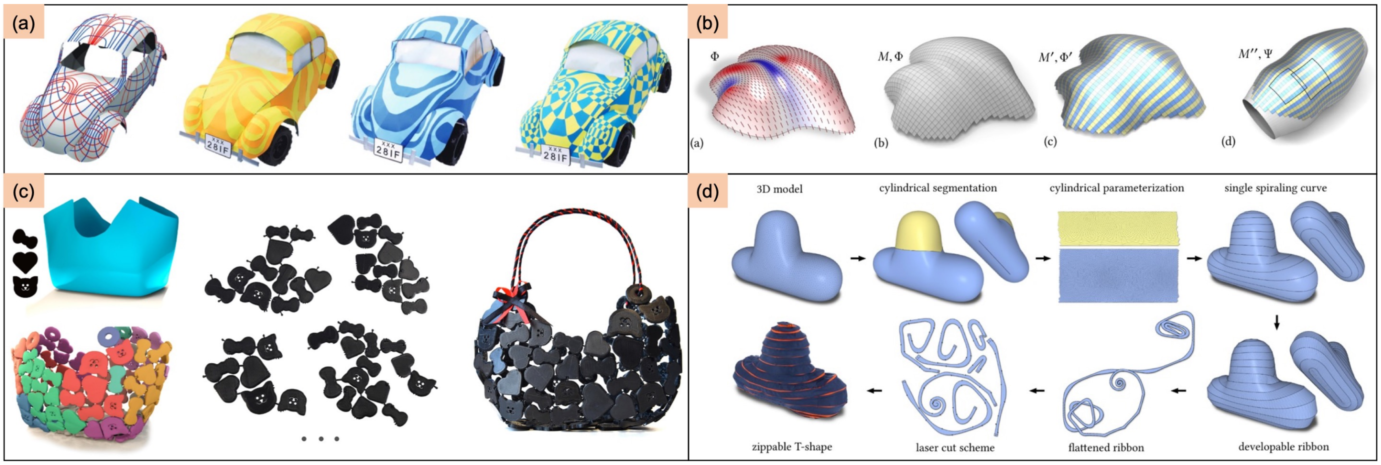


Fig. 4. (a) Construction of principal strips for woven 3D shapes by principal curvature [32]. (b) Construction of quad meshes by a killing vector field [37]. (c) Tessellation of irregular polygons on the target surface [49]. (d) Custom-cutting by using the spiral curve rule [59].

operations of the sequential strategy (see Fig. 2c). Custom-cutting involves users providing specific operations through interaction, allowing operators to split 3D shapes according to their ideas and experiences. Based on collected studies, custom-cutting can be categorized into five types: cross-section-based, loft-based, feature-based, rule-based, and experience-based.

Cross-section-based involves horizontal cutting based on the cross-sections of columnar 3D shapes, such as cutting of 3D garments based on the characteristics of human cross-sections [50], and unfolding a vase through cross-section cutting [51]. Loft-based focuses on generating developable surfaces through lofting with two spatial curves [52]–[55]. Feature-based involves cutting shapes according to obvious geometric features [56]–[58]. Rule-based involves segmenting shapes into special pieces through specific rules, often resulting in creative outcomes. For instance, a cutting method by using spiral curves is employed for unfolding closed shapes [59] (see Fig. 4d). Experience-based refers to the segmentation of 3D shapes by the designer's expertise, leading to a specific design style, such as 3D tailoring [41], [60]–[63]. Note that after defining specific cutting paths, these surfaces often need to be converted into developable surfaces. Depending on the cutting path type, cross-section-based, loft-based, and rule-based paths that form strips can model developable or ruling surfaces [50], [52]–[55], [59]. However, feature-based cutting paths present challenges for directly modeling developable surfaces and often require developability optimization to achieve them [56].

3.3 Developing

After cutting operations are used to form pieces such as strips or polygons, developing methods convert these pieces into 3D developable surfaces or planar pieces in space to approximate the target surface. Therefore, if fabrication is required, these generated developable surfaces or planar pieces need to be mapped onto a 2D domain. According to the collected studies, one approach is **developable surface modeling** for modeling the target shape directly using 3D developable surfaces generated by cut curves, while another is **planarity optimization** for focusing on planarizing polygonal pieces while preserving the target polygonal mesh as much as possible. Additionally, yet another is **developability optimization** for transforming the entire target shape into 3D developable or quasi-developable surfaces.

Developable Surfaces Modeling. Developable surface modeling refers to directly using developable surfaces to build a shape to approximate the target shape. This operation is usually interactive. Tang et al. [64] proposed a user-driven modeling method where the user guides spline surface projection to approximate the target surface. Ion et al. [26] approximated the target surface using piecewise developable surfaces based on the "Discrete Orthogonal Geodesic Nets" method proposed by Rabinovich et al. which is used for interactive modeling of developable surfaces without the need for computing global geodesic lines. Additionally, existing studies model developable surfaces directly using custom curve frameworks, such as loft-based curves [50], [52]–[55] and developable strips generation by spiral curves [59] (see Fig. 4d).

Planarity Optimization. For a polygonal mesh, if each mesh face is a non-triangular face, such as the quadrilateral or hexagon, these faces are often non-planar, meaning the polygon's vertices do not lie on a single plane. The aim of the planarity optimization is to obtain planar polygonal faces while preserving the overall shape of the target surface, i.e., planar polygonal mesh (see Fig. 2a and b). The planarity optimization can be framed as a geometric optimization task. The re-meshed surfaces is donoted as $M = (V, E, F)$, V, E, and F represent the vertex set, edge set, and face set of M, respectively. We denote the vertex of M as $v_i, i \in V$, and the unit normal of the face as $n_k, k \in F$. Therefore, the planarity condition can be expressed as: $n_k^T(v_i - v_j) = 0$, where $(i,j) \in E(f_k)$ is an edge of the polygonal face f_k. Additionally, the planarity condition can be converted into the energy function as follows:

$$E_{plan} = \sum_{k \in F} \sum_{(i,j) \in E(f_k)} (n_k^T(v_i - v_j))^2 \tag{1}$$

To keep the overall shape of the target surface S as unchanged as possible, a common idea is to constrain the vertices onto S as much as possible. The closest point on S for a vertex v_i is $\tilde{v}_i$ with corresponding normal $\tilde{n}_i$, the

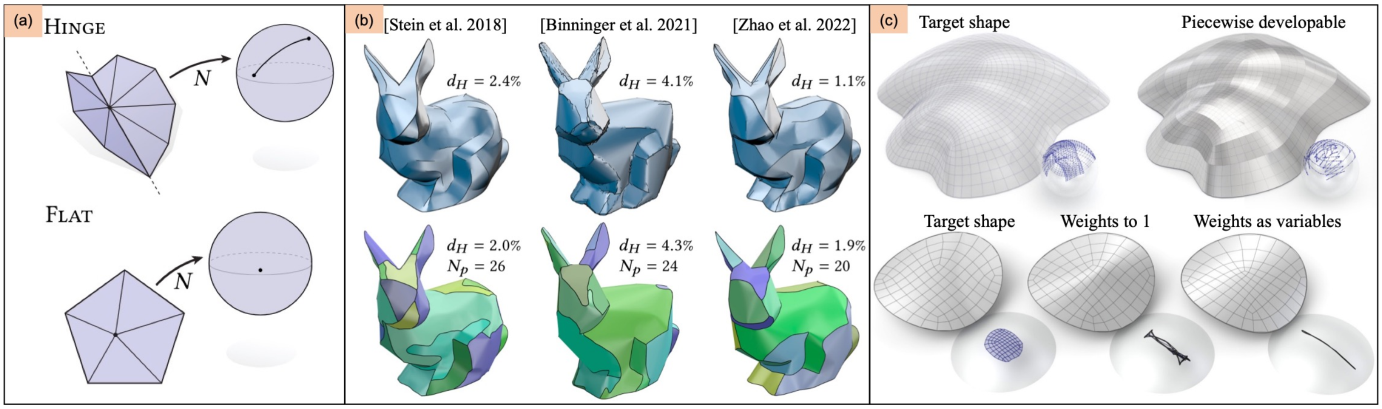


Fig. 5. Developability optimization: (a) Gaussian image of the hinge-like structure [69]. (b) Different piecewise developable results, left [69], middle [73], and right [74], where N_p represents the number of patches, $|d|_H$ represents the the two-sided Hausdorff distance (with respect to the diagonal length of the bounding box of the input triangular mesh). (c) Discrete developability for quad meshes equipped with vertex weights [76].

energy function is expressed as follows:

$$E_{close} = \sum_{v_i \in V} ((v_i - \tilde{v}_i) \cdot \tilde{n}_i)^2 \tag{2}$$

Therefore, the composite energy function can be expressed as: $E = \omega_{plan} E_{plan} + \omega_{close} E_{close}$. Existing research employ the common idea to construct planar quad (PQ) meshes [65], [66] and polyhedral patterns [48]. Similarly, Bhooshan et al. [67] construct a planarity energy function by measuring the distance of each vertex of the face from the best-fit plane. The initial best-fit plane is created by calculating the sum of the normals of each triangle in the face, where each triangle is formed by an edge of the face and the average position of all vertices of the face. Additionally, if only for the quad mesh, the distance between the face diagonals can be used as the planarity measure [68].

Developability Optimization. Developability optimization involves directly optimizing target surfaces into a shape consisting of 3D developable or quasi-developable surfaces (Fig. 2d) by establishing relevant developability metrics. The Gaussian image can be used for visualizing developability. The principle refers that each unit normal of a 3D surface as a point are mapped onto a unit sphere S^2, where the distribution of points reflects the local flatness of the target shape. A point on the sphere indicates local planarity, arcs represent developable surfaces, curves denote ruling surfaces, and regions signify non-developable surfaces. Consequently, related research often establishes specific developability metrics based on surface normals, such as vertex or face normals of a mesh surface. For example, Stein et al. [69] proposed a method for smooth developability, and the method optimizes the normal direction of each vertex of the ”vertices star” (a polygon with a vertex at its center) into two directions, forming a hinge-like structure. i.e., each ”vertices star” can be divided into two regions $F_1, F_2 \subset F$. The average normal of the triangle unit normals N_σ in region F_p can be expressed as: $\overline{N}_p := \frac{1}{n_p} \sum_{\sigma \in F_p} N_\sigma$. Therefore, an energy function can be built through the deviation between N_σ in each region F_p and $\overline{N}_p$, which is expressed as:

$$\begin{aligned} \pi(P) := & \sum_{p=1,2} \frac{1}{n_p} \sum_{\sigma \in F_p} |N_\sigma - \overline{N}_p|^2 \\ = & \sum_{p=1,2} \frac{1}{n_p^2} \sum_{\sigma_1, \sigma_2 \in F_p} |N_{\sigma_1} - N_{\sigma_2}|^2. \end{aligned} \tag{3}$$

where $\sigma \in F$ denotes the triangle. This method narrows the width of the shape's discrete point range on the Gaussian image (see Fig. 5a and b left).

Additionally, Zeng et al. [70] constructed a developability metric, which computes the difference between triangles' orientation and target orientation, then the measure is minimized by least squares method to obtain a quasi-developable surfaces by a mesh surface as input. Bhooshan et al. [67] employed a discrete measure of Gaussian curvature that is proportional to the sum of the surrounding corner angles at each internal vertex of the mesh. When the sum of angles equal to 2π, the vertex has zero Gaussian curvature. Therefore, the difference of the 2π and the sum of angles is minimized by the dynamic relaxation method. This method is used to unfold a planar polygonal mesh into a piece without cuts. Gavriil et al. [71] introduced a method for increasing the developability of a surface through an optimization algorithm that thins the Gaussian image of face normals, which forms a quasi-developable surface. Sellan et al. [72] proposed a method for developable approximation through rank minimization. This method utilizes height fields representing surfaces and minimizes the rank of the Hessian matrix. Binninger et al. [73] gradually deform the target shape into piecewise developable surfaces by locally thinning the Gaussian image. The fundamental idea involves considering local neighborhoods on the Gaussian image and approximating each neighborhood with arcs, moving the central point of each neighborhood onto the corresponding arc (see Fig. 5b middle). Zhao et al. [74] constructed an “edge-oriented” developability as a degeneracy condition for Gaussian image, which can realize the piecewise developable approximation of target shapes. The workflow has three stages: deformation, segmentation and refinement. Compared with previous studies [69], [73], this study can generate less patches and has less distortion, but it has high computational cost (see Fig. 5b right). For quad mesh developability, Jiang et al. [75] proposed a method to define the isometric mapping of discrete quad meshes via properties of a checkerboard pattern inscribed in the original mesh. The checkerboard patterns generated by inserting midpoints of edges always form parallelograms. Inza et al. [76] proposed a discrete developability criterion based on rank-deficient second fundamental form, which is applied to discrete developability of quad meshes equipped with vertex weights. The criterion involves assigning contact elements to the faces of meshes and ruling vectors to the edges, which collectively yield a developability condition per face. The method is only suitable for quad meshes and requires experience to choose weights (see Fig. 5c).

During the process of some developability optimization methods, the target surface is optimized into a single developable surface with creases, which can be further converted into cut seams. However, creases are often uncontrollable.

3.4 Flattening

After obtaining shapes that consist of 3D developable surfaces or planar pieces, such as piecewise developable surfaces or planar polygonal meshes, the flattening operation is required to create 2D patterns for fabrication. Depending on the type of target surface, the developing operation yield different shapes, leading to a variety of suitable flattening techniques. According to collected studies, we classify three methods including **isometric mapping** for flattening 3D developable surfaces or planar pieces in space, **kirigami** for flattening planar polygonal meshes by constructing special kirigami structures, and **spanning tree** for flattening planar polygonal meshes based on graph theory (see Fig. 6a).

Isometric Mapping. For 3D developable surfaces, flattening does not cause shape distortion, which is an isometric mapping. From the perspective of differential geometry, points in an infinitesimal neighborhood on a 3D surface are mapped to corresponding positions on a 2D plane using the Jacobian matrix J, which can be decomposed using Singular Value Decomposition (SVD):

$$J = U\Sigma V^T = U \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \\ 0 & 0 \end{pmatrix} V^T \tag{4}$$

where σ_1 and σ_2 are singular values. When $\sigma_1 = \sigma_2 = 1$, J is an orthogonal matrix, representing a rigid transformation. Therefore, we can initialize a flattened shape by corresponding methods such as direct projection or Tutte's theorem [95] to obtain the initial Jacobian matrix and its singular value for each vertex. During the flattening process, we can gradually optimize these singular values to 1 by minimizing an appropriate objective function to achieve isometric mapping. A common objective function that flattens the shape as rigidly as possible [84] can be used to measure shape distortion:

$$E_{iso} = (\sigma_1 - 1)^2 + (\sigma_2 - 1)^2 \tag{5}$$

Additionally, from an intuitive geometric perspective, existing studies utilize the property of edge length preservation in the target surface to construct the objective function. Each vertex X_i of the target mesh M is mapped onto a 2D plane, where the objective function can be constructed by considering the length loss of the mesh edges as follows:

$$E_{len} = \sum_i \sum_{j \in \mathcal{N}(i)} w_{ij} ||(X_i - X_j) - R_i(X_i - X_j)||^2 \tag{6}$$

where $\mathcal{N}(i)$ is the set of vertices adjacent to X_i. The R_i is a mapping that transforms edges onto the 2D domain. The weight w_{ij} represents the cotangent weight. As the mesh is composed of triangles, this objective function is often used as a measure of distortion for isometric mapping. Some studies use the objective function to achieve isometric flattening, such as [63], [75], [85].

kirigami. For planar polygonal meshes, an approach is to construct special kirigami structures flattened into a piece for fabrication. Kirigami as a generalizing origami refers to the papercutting art, combining folding and cutting techniques, including creating holes. A common approach is to use a single sheet of paper to form specific 3D shapes, such as straight-line origami [96], [97], curved-line origami [98], [99], and "pop-up" art [100], [101]. These studies focus on transforming a 2D pattern into a 3D shape. In contrast, we address the inverse problem: constructing kirigami structures to unfold target 3D shapes into 2D patterns.

A core idea is configuring hinge structures within the target polygonal mesh to create the desired kirigami structure. Given a target polygonal mesh with planar faces, additional hinges are introduced between adjacent faces, forming creases, which can be unfolded into a piece [85], [86] (see Fig. 6b). While this method has been extended to 3D printing on stretchable materials [102], it's important to note that using an elastic fabric to connect adjacent mesh faces does not produce a developable structure.

After forming the kirigami structure with the target polygonal mesh as input, the structure can be flattened without distortion based on isometric mapping. However, since the kirigami structure has folding hinges, flattening it directly would cause the faces to overlap. To obtain better flattening results, it is necessary to add additional objective functions to avoid overlapping, such as increasing a fairness objective function. Specifically, for a regular mesh, there are three families polylines. Therefore, the objective function can smooth the mesh and prevent mesh overlapping by optimizing these polylines [37], [68], [75], [85], which can be expressed as:

$$E_{fair} = \sum_{c \in C} \sum_{(i,j,k) \in c} ||b \cdot (X_i - 2X_j + X_k) - (\bar{X}_i - 2\bar{X}_j + \bar{X}_k)||^2 \tag{7}$$

where c denotes the index set of the vertices of a polyline, and $c \in C$. $\bar{X}_i$ represents the position after deformation. The parameter $b = 0$ represents absolute fairness, while $b = 1$ represents fairness relative to the input mesh.

Spanning Tree. For planar polygonal meshes, another approach is to transform these planar faces onto the 2D domain directly through rigid coordinate transformations. However, the assembly of a large number of pieces is cumbersome. To reduce the number of the piece, one approach is to construct spanning tree based on graph theory. In graph theory, a spanning tree is a connected subgraph that contains all the nodes of the graph but with the least number of edges. For a mesh, it can be transformed into a dual graph, where mesh faces $f_i \in F$ serve as nodes connected to adjacent mesh faces $f_j \in \mathcal{N}(i)$, forming corresponding edges. Spanning trees can be used to represent the connectivity of mesh faces, When a root node is specified (which can be randomly specified), the adjacent faces can be gradually tiled onto a 2D plane according to the connection order in the spanning tree.

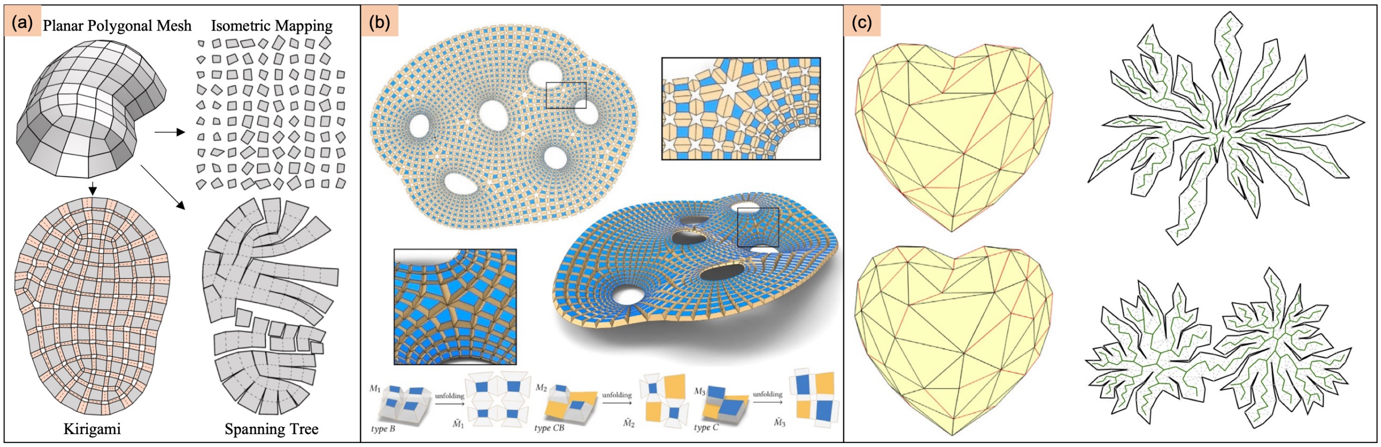


Fig. 6. (a) Demonstration of three flattening methods for a shape: mapping each mesh face to a 2D plane using isometric mapping; transforming the target shape into a kirigami structure to flatten it into a single piece; and flattening the shape using a spanning tree method. (b) Flattening the kirigami structure by a planar polygonal mesh adding hinges [85]. (b) Flattening a planar polygonal mesh by minimum spanning tree [87].

Some methods for spanning trees are available for non-regular meshes. Chandra et al. [42] obtained a minimum spanning tree based on greedy algorithms. Xi et al. [87] grouped nodes through clustering to generate a spanning tree (see Fig. 6c). Kim et al. [88] extended a genetic-based algorithm to optimize a spanning tree for convex shell mesh. For regular meshes, some studies utilize the connection relation of the four-edge data structure to generate intersecting assembly structures [89], [90]. Leung [91] generated long chains by entering regular meshes, which are used for weaving into surfaces.

Note that planar polygonal meshes are flattened using the minimum spanning tree method, which often encounters the issue of overlapping. Therefore, it is necessary to implement partitioning to avoid this problem (see Fig. 6a).

3.5 Auto-Cutting

In addition to using sequential strategies to create 2D patterns, there are methods that directly optimize target surfaces into multiple 2D patterns by integrating strategies. We refer to these methods as auto-cutting, with the core idea being the computation of cut paths during the flattening process to minimize distortion of the target surface. By automatically computing the cut paths, auto-cutting eliminates the need for specialized cutting knowledges from the user. For example, Poranne et al. [77] proposed a joint method for simultaneously optimizing cuts and geometric distortion. This method models UV mapping by parameterizing each triangle and employs attraction energy to encourage continuity over matching edge pairs. Therefore, the method consists in solving the following optimization problem:

$$\min_{\mathbf{X}} \mathrm{E}(\mathbf{X}) = \min_{\mathbf{X}} (1-\lambda)\,\mathrm{D}(\mathbf{X}) + \lambda\,\mathrm{S}(\mathbf{X}) \tag{8}$$

where D is a distortion objective that measures triangle distortion, S is a separation objective that measures an edge separation, and the parameter λ controls the balance between the two objectives. As the algorithm converges, these edges eventually become seams or regular edges. The method supports interactive cutting and vertex movement (see Fig. 7 middle and right).

Additionally, Julius et al. [78] introduced the D-Charts mesh cutting algorithm, which divides the mesh by calculating the deployability measure of the mesh surface. While this algorithm reduces distortion, some detailed features may not be preserved. Lévy et al. [79] proposed the Least Squares Conformal Maps (LSCM), an automatic method for generating texture atlases. It decomposes the model into patches homomorphic to disks using a feature detection algorithm, parameterizes each patch, and packs the unfolded patches into the texture space. This method reduces angular distortion, prevents triangle flips, and results in a texture with a free boundary, but it exhibits higher area distortion. Sheffer proposed a curvature-based mesh segmentation method [80], which calculates positions with high curvature as cutting points. Subsequently, cutting lines are generated by applying a minimum spanning tree algorithm on the corresponding graph of the target mesh. Sorkine et al. [81] introduced an automatic parameterization method that controls distortion. It starts by randomly selecting a triangle as a seed and unfolds it iteratively onto a 2D plane (see Fig. 7 left). The process selects the best vertex in each iteration to minimize distortion, embedding vertices only if they cause distortion within a predefined threshold. The method stops when no vertices meet this criterion, and a new seed triangle is chosen for further iterations. However, the method does not provide an optimal solution and lacks explicit control over seam positions or lengths. Li et al. [82] further proposed a joint optimization algorithm called OptCuts. Users provide the distortion bound and initial seams, and the algorithm automatically seeks the minimum length of seams within the specified distortion bound. Zhao et al. [83] introduced a method for computing piecewise developable approximations for triangular meshes. The key approach is the utilization of a genetic algorithm to optimize a combinatorial fitness function, which incorporates various factors such as the approximation error, the number of patches, the length of patch boundaries, and penalties for small patches and narrow regions within patches. The main challenge in this method is evaluating the approximation error of the fitness function. To efficiently measure distortion, the authors employ conformal mapping methods without explicitly generating developable shapes to reduce computational cost.

Auto-cutting is essentially a form of surface parameterization, but some shape distortion may still occur. Increasing the cutting weight can help reduce this distortion, but it

TABLE 2
Features of different physical modeling methods.

Physical Modeling	Merits	Defects	Process	Examples
Cutting	·Fast production speed ·Low cost of production ·Wide selection of materials	·Tedious assembly	· (1) Extract contour curves and dotted lines · (2) Convert curves into machine instructions · (3) Place a whole piece material to complete cutting	[90]
Casting	·Fast production speed ·Reused	·More work procedures	· (1) Design flat molds meeting features of easy demolding · (2) Pour liquid materials to molding · (3) Remove molds to fabricate flat panels	[57] [58]
3D Printing	·Suitable for mass custom production	·Limited choice of materials	· (1) Convert 2D patterns into flat solid structures · (2) Slice structures into G-code · (3) Control over the 3D pirnter to fabricate flat structures	[49] [105]
Knitting	·Low cost of production ·Suitable for flexible materials production	·Limited choice of materials ·Unavoidable error	· (1) Convert 2D patterns into pixel images with specific colors · (2) Convert pixel images into stitch instructions · (3) Control over the CNC machine to fabricate fabrics	[59] [30]
Folding	·No need to combine pieces	·Weak stability ·Tedious folding	·Fold the flat piece along a crease or dotted lines	[85]
Joint	·Strong connection	·Tedious assembly	·Joint seams of adjacent flat pieces with connection structures	[90]
Woven	·Not require additional fixation	·Tedious assembly	·Weave the strips in a specific order	[104]

also increases the difficulty of subsequent assembly. Additionally, the lengths of the cut seam pairs between the two patches may differ, potentially leading to assembly failure. Furthermore, the shape of the cut seam remains uncontrollable.

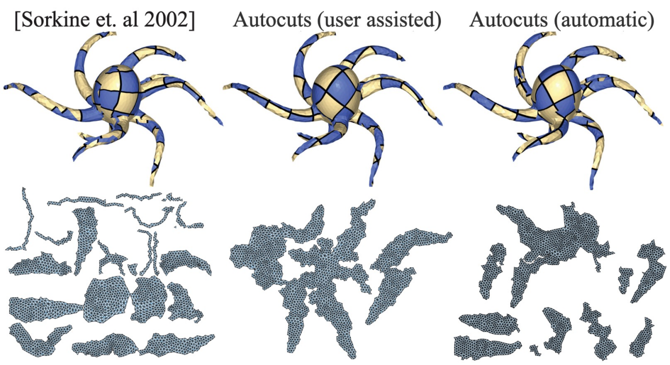


Fig. 7. Auto-cutting by calculating cut paths, left [81], middle and right [77].

4 Physical Modeling

Physical modeling involves transforming digital models into physical shapes in the fabrication process. The goal of Section 3 is to generate either one or more 2D patterns. Consequently, the goal of the physical modeling is to create physical shapes employing **fabrication** and **assembly** methods (see Fig. 8 and Table 2).

4.1 Fabrication

Fabrication refers to the process of creating components or structures by manipulating raw materials through various techniques. It encompasses a wide range of fabrication methods aimed at shaping materials into desired forms. According to the fabrication methods used in the collected studies, these fabrication methods for making flat structures include **cutting**, **casting**, **3D printing**, and **knitting**.

Cutting. Cutting as one of the subtractive manufacturing is one of the most commonly used fabrication methods in sheet materials. Cutting refers to the process of dividing a sheet along specified tool-paths to form flat patterns. It offers fast production, cost-effectiveness, and a wide range of material choices. The cutting method can be employed to fabricate 2D patterns using a CNC cutting machine. The basic process is as follows: (1) extract the contour curve of the digital 2D pattern. If there is a crease within the 2D pattern, it can be represented by dotted lines; (2) convert these curves into instructions for controlling the CNC cutting machine, such as G-code; (3) place a whole piece of flat material into the CNC cutting machine to complete the cutting along the specified path. However, assembling a 3D shape using planar materials can be a tedious task, especially when a large number of pieces need to be cut.

Cutting is widely employed in the fabrication of shapes formed by developable surfaces, utilizing various materials. Prototyping through paper cutting is employed in several studies, including [32], [56], [74], [85], [87]–[91] (see Fig. 8a). Metal cutting fabrication methods are discussed in works such as [67], [106], [107]. Additionally, fabrication techniques involving hard materials cutting [108] and cloth cutting [50], [58]–[60], [70], [109] have also been explored.

Casting. Casting is a widely used fabrication method for producing large quantities of flattened panels with large scale size. The basic process of a casting method is as follows: (1) design flat molds based on desired 2D patterns, ensuring features like draft angles or removable sections to facilitate demolding; (2) pour liquid materials, like metal, plastic or concrete, into molds, let materials cool or solidify; (3) remove the mold to reveal the finished flat panels. Molds can be reused multiple times to save costs of fabricating large size of surfaces. However, additional steps are required for fabricating and removing molds.

Casting is often used in the production of panels in building [39], [68], [92]–[94], [110], [111]. Molds can be constructed by developable surfaces, which reduce the fabricating difficulty of molds [57], [58] (see Fig. 8b).

3D Printing. 3D printing technology is widely employed in research and prototyping due to its rapid molding speed and the ability to easily produce complex surfaces. It employs multi-layered planar paths to fabricate 3D shapes. However, general 3D printing techniques often require support structures when fabricating curved forms, leading to cumbersome post-processing. Although existing research introduces multi-axis 3D printing techniques for fabricating complex shapes without support structures [112]–[116], challenges such as complex tool-path planning and high equipment costs remain. Deforming into 3D shapes by fabricating planar structures using general 3D printers remains

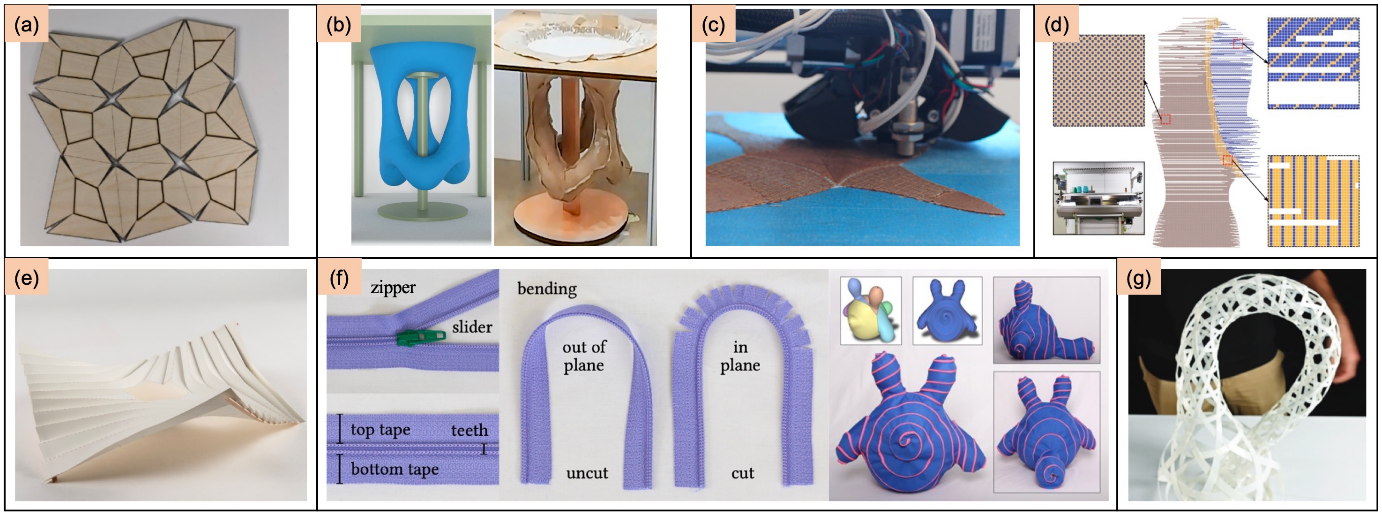


Fig. 8. Selected samples of physical modeling are figures of fabrication (a-d) and figures of assembly (e-g). (a) Cutting [85]. (b) Casting [58]. (c) 3D printing [103]. (d) Knitting [30]. (e) Folding [22]. (f) Joint [59]. (g) Woven [104].

a low-cost alternative. The basic process of 3D printing for fabricating flat structures is as follows: (1) convert 2D patterns into flat solid structures using a CAD software; (2) slice solid models into G-code using a slicing software; (3) load the G-code into the 3D printer to produce flat structures.

The production process of complex shapes uses printed flat structures without supported structures, which can reduce material waste, shorten printing time, and simplify post-processing [49]. Several studies have utilized this method, such as [49], [102], [105], [117] (see Fig. 8c).

Knitting. Knitting is a fabrication method that utilizes threads to create a 2D plane through knitting. The extensive utilization of CNC knitting machines has expedited and economized production, especially for flexible materials. The basic process of a CNC knitting technique is as follows: (1) convert the 2D pattern into a pixel image (such as a BMP file), assigning pixel colors based on specific structures and requirements to correspond with different stitch types; (2) import the pixel image into knitting software to generate stitch instructions; (3) use these stitch instructions to control the CNC knitting machine to produce the fabric. However, knitting is primarily applicable to thread-like materials, and discrepancies between real and digital shapes are inevitable.

Knitting is widely used in the fabrication of the shapes formed by developable surfaces [50], [58]–[60], [70], [109]. In addition, there are also knitting processes based on geodesic principles that directly form surfaces without cutting and sewing. [30] (see Fig. 8d).

4.2 Assembly

Assembly refers to forming the produced flat pieces into the target shape. From the studies of collections, assembly can be classified into three types: **folding**, **joint**, and **woven**.

Folding. Folding is a way to transform a plane into a 3D shape by setting creases with straight or curved lines. Unlike methods requiring the combination of multiple parts, folding involves the deformation of a single flat piece. This approach is suitable for specific folding structures [22], [105] (see Fig. 8e), and kirigami structures that fold flat structures to form target shapes without partitions [85], [86] (see Fig. 6b). The advantage of folding is that it eliminates the need to combine multiple components. However, its structural stability is weak, and the folding process can be tedious.

Joint. Joint involves combining multiple components through specific connections. Various joint types exist based on the material used, such as gluing or splicing for paper, welding for metal, and sewing for soft materials like cloth. Additional methods include zippable [59] (see Fig. 8f) and magnetic connections [90]. Joint offers the benefit of a robust connection between parts, but the assembly process is often intricate. While the boundaries of developable surfaces are mutually congruent, it is often necessary to create overlapping auxiliary surfaces to reinforce connections. Therefore, connection methods that involve auxiliary surfaces should avoid excessive curvature within the boundaries of developable surfaces to prevent assembly difficulties [59].

Woven. Weaving is a method used to construct shapes by interlacing linear materials, as demonstrated in several studies [32], [91], [104] (see Fig. 8g). It eliminates the need for additional fixation and offers high stability. However, the assembly process is complex, and discrepancies between real and digital shapes are inevitable. This method involves approximating target shapes using strips that are formed by regular grids.

In conclusion, physical modeling plays a crucial role in transforming digital models into real objects. Appropriate fabrication and assembly methods should be selected based on different design outcomes and material properties. By leveraging the advantages of developable surfaces and fabrication techniques, the ”design to fabrication” approach can effectively facilitate rapid construction and the development of new materials.

5 Interactive Assistance

This section focuses on the relevant interactive assistance in the fabrication process. In the digital modeling process, some cutting operations are not well suited to certain characteristics of the shape. For example, they may produce irregularly shaped pieces, resulting in complex production and assembly. Interactive cutting allows for more controlled

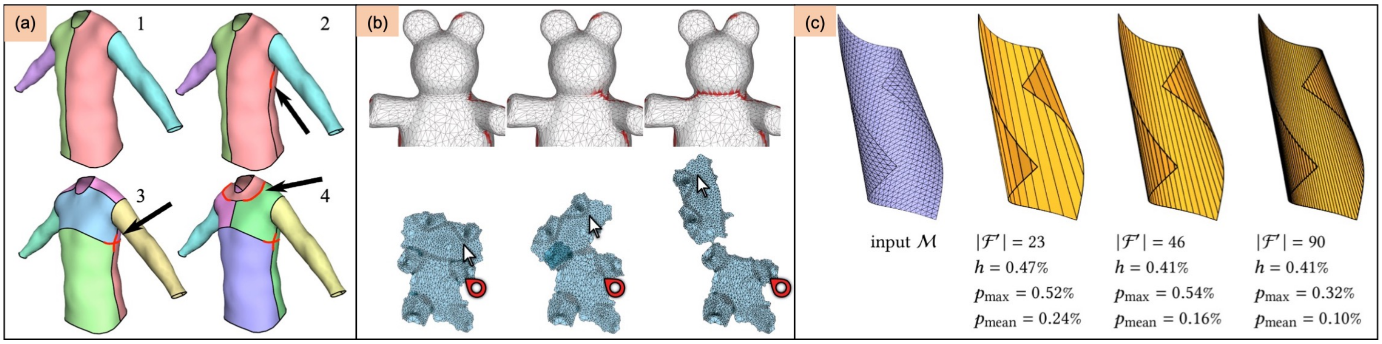
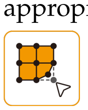


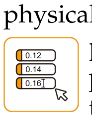
Fig. 9. Selected examples of interaction: (a) Cutting for segmentation of clothes by guidance of users [41]. (b) Unfolding for surfaces by dragging [77]. (c) Parameterizing for optimization by setting parameters [34].

results. Additionally, when surfaces are flattened, many problems occur during the optimal computation, including extended iteration times and mesh faces overlapping. The global optimal solution is also difficult to obtain. Interaction can guide the optimization direction through visual observation, reducing the need for extensive iterations and enabling smoother unfolding through interactive dragging and parameter adjustments. According to collected studies, interactive assistance can be classified into three interactive operations: **cutting** for custom drawing of curves on shapes to form seams, **unfolding** for dragging deformation, and **parameterizing** for control and adjustment of the parameters.

Cutting. Cutting is the interactive process of cutting a shape by users selecting edges or drawing curves. For instance, Real-time generation of unfoldable surfaces from curves drawn by the user on a digital board [52], dividing a shape into many surfaces by drawing feature lines on the original 3D shape for making sky lanterns [56], drawing structural frame lines in a shape for segmentation [57], interactive segmentation [77], cloth segmentation by user guidance [41], [118] (see Fig. 9a). Cutting is mainly used for segmenting a 3D shape into multiple surfaces. Some auto-cutting methods can automatically split 3D shapes, but the user-interactive approach is more appropriate for the control of details.



Unfolding. Unfolding involves users dragging a digital model to prevent the flat pieces from overlapping and construct folding structures through the graphical user interface. For instance, generating origami structure by dragging mesh faces [64], [86] and interactive vertex movement to eliminate overlap [77] (see Fig. 9b). Unfolding is a very free interactive operation to control the deformation for surfaces, similar to deforming objects in the physical world.



parameterizing. This term doesn't refer to surface parameterization, instead, it signifies users interactively manipulating algorithm's parameters to influence the developable approximation, thereby altering the flattening results. Some developable approximation algorithms can obtain different optimization results by adjusting parameters [26], [34], [73], [74] (see Fig. 9c). In addition, parameterizing is also utilized to control and adjust CNC equipment used in physical modeling.

In conclusion, interactive assistance plays a crucial role in the fabrication process, with many studies leveraging interaction to simplify complex tasks and enhance controllability and efficiency.

6 Design Applications

This section focuses on applications of the fabrication process using developable surfaces approximiation. Depending on the field of application from collected studies, we classified them into six application directions: **architecture**, **industrial products**, **arts and crafts**, **garments**, **mechanical materials** and **data physicalization**.

Architecture. Developable surfaces find extensive application in architecture, particularly in large-scale buildings where component construction and assembly are essential due to the vast spatial scale. Complex curved surface architecture typically comprises numerous planes or simple curved surfaces. The usage of developable surfaces in curved buildings can be categorized into four main directions: facade, shell, bricklaying, and frame. A facade serves as the exterior of a building, typically subdivided into numerous similar flat or simply curved panels. Techniques such as clustering are employed to reduce the number of panel types [65], [68], [71], [92]–[94]. Alternatively, The surface can be segmented into multiple strips, which can then be approximated as developable surfaces. After fabricating these flattened surfaces, they are bent to construct the target form [27]. Shell is a self-supporting building form that is widely used due to its excellent force structure, space with large spans, and graceful curved shapes. Shells can be constructed using discrete meshes [121], or by combining numerous developable surfaces [31], [71], [119], [122] (see Fig. 10a). This method offers greater reduction of assembled parts compared to discrete mesh approaches. Frame maintains the basic shape and support of the building, which are usually made by combining rod-like materials, Frames can be formed by meshing shapes [44], [66], [67], [106], [110], [123]. Frames can also be formed by combining long, elastic materials [28], [29]. Bricklaying is an ancient yet still widely used construction approach that uses prefabricated, uniformly sized materials. It can form not only flat surfaces but also curved surfaces [39] and rich textures. This process is also the topic of extensive research in digital construction.

Industrial Products. For industrial products, these products, characterized by functionality and mass production, adhere to standardization. Utilizing developable surfaces in production can streamline fabrication processes and impact the design aesthetics of industrial goods [49], [59]. For vehicle design, as the shapes are often complex surfaces. Generating shapes for automobiles and ships uses the method of combining many developable surfaces, which can reduce

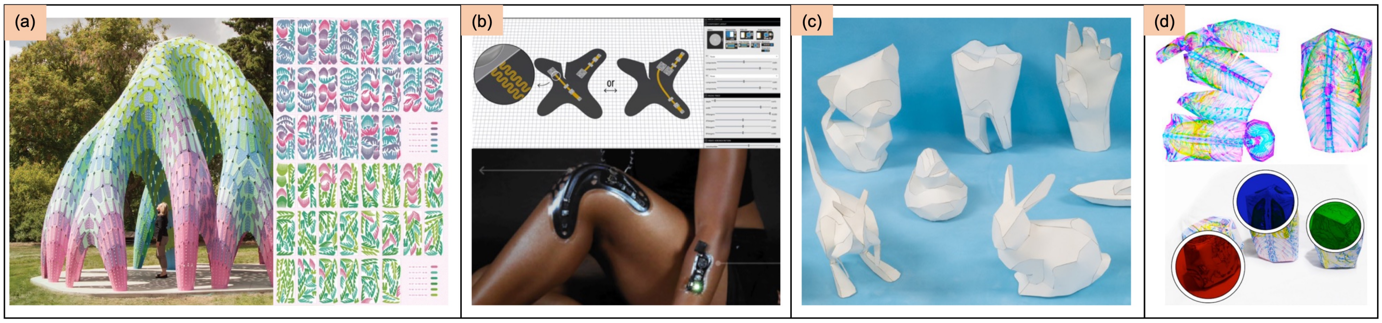


Fig. 10. Selected samples of design applications: (a) Shell construction [119]. (b) Wearable device [61]. (c) Paper folding [74]. (d) Anatomical physical visualization [120].

the manufacturing difficulty and cost [52], [55]. A new design style is developed through the developable surface method. For instance, the zipit bag features a strip with a spiral structure, enabling the creation of various cartoon forms. In addition, flexible products can be fabricated by fitting complex shapes with developable surfaces, such as fabric toys [59] and wearable devices [61] (see Fig. 10b). Chairs are frequently designed with curved shapes to enhance comfort. Some studies utilize developable surfaces to craft curved surfaces that conform to sitting posture, resulting in graceful shapes [64], [117].

Arts and Crafts. Arts and crafts involve creating physical forms with aesthetic appeal through various processes and materials. Compared with painting as fine art, arts and crafts are constrained by the materials and techniques used, leading to distinct artistic styles. Shapes crafted through skillful bending and folding often exhibit polygonal and regular features [124]. Some of the research is inspired by paper folding [74], [85]–[88], [90], [102](see Fig. 6b, c, and 10c). In addition, piecewise developable surfaces are also used for inflatable structures, such as inflatable dolls [57]. In addition, some types of forms and modeling methods are enhanced by inspiration of developable surfaces method, such as weaving art with free-form surfaces [104], sky lanterns with free-form surfaces [56], sculpture with fabric mold [58], and peeling art by isometric mapping [125]. These art forms based on developable surfaces demonstrate creativity through the rational use of materials.

Garments. Garments typically comprise multiple flat fabrics, and manufacturing directly influences the garment design process. Various 3D tailoring methods leverage the bending and folding of developable surfaces in garment design [41], [118]. Designers present the design language by fully applying the characteristics of the 3D tailoring to achieve artistic shapes. In addition, approximating 3D shapes using piecewise developable surfaces are widely used for garment design and fabrication [63]. Improving the comfort of garments is also an important goal of garments design. The piecewise developable approximation methods can reduce the local stretching and shearing of the garment and make it more fit the body [60], [70]. In addition, some 3D fabric fabrication methods use distance field method to reduce wrinkles in garments, which allows the fabric to be produced directly as the 3D form, and sewing is unnecessary [30] (see Fig. 8e).

Mechanical Materials. For mechanical materials, the bending and folding properties of developable surfaces are used in the study of deformable structures, such as 4D printing materials [105], kirigami structures [85], [86] and design of deformable structural materials [107], [108]. Research employing developable surfaces offers innovative avenues for designing new materials and interactions.

Data Physicalization. Developable surfaces as a type of physical structure provide effective methods for data physicalization. A study explored anatomical physical visualization [120] (see Fig. 10d). The use of developable surfaces has more advantages than static physical objects, such as simple fabrication and dynamic presentation of data in the real world by deforming shapes between 2D and 3D (likes the flattening and folding of the world map AuthaGraph).

In conclusion, a wide range of applications demonstrates the potential of developable surface methods, both as a low-cost manufacturing to optimize production processes and as a way to develop new design processes and design styles. From the above applications, Innovative ideas must consider developable surface characteristics and employ suitable fabrication methods.

7 Discussion

In this section, we discuss the challenges retained in the existing research and propose directions for future research.

Quality of Cutting. The quality of cutting impacts subsequent flattening, fabrication, and assembly. Cutting can be evaluated by factors such as whether the parts are regular, similar, and easy to flatten. Although there are some automated cutting methods, they often lead to irregular results, increasing the difficulty of subsequent fabrication and assembly processes [73], [74]. When dealing with regular shapes, manual cutting can yield better results based on experience, but handling complex shapes manually remains a challenge [59]. Furthermore, by collecting and analyzing geometric properties, it is possible to explore general cutting methods for complex shapes. On the other hand, developing new fabrication methods can make cutting easier.

We also try to collect studies of shape cutting through AI in the field of developable surfaces, but there is no relevant research at present. Since shape cutting can be regarded as the classification of mesh faces, some studies on AI-based mesh segmentation can also be followed. For example, some studies provide basic topological meshes through mesh simplification by AI, facilitating the subsequent processes of mesh segmentation and parameterization, e.g., "QEM-based mesh simplification" [132], and "MeshCNN" [133]. Research also implements semantic-based shape segmentation by training mesh labels [134]–[138]. Other studies are on shape

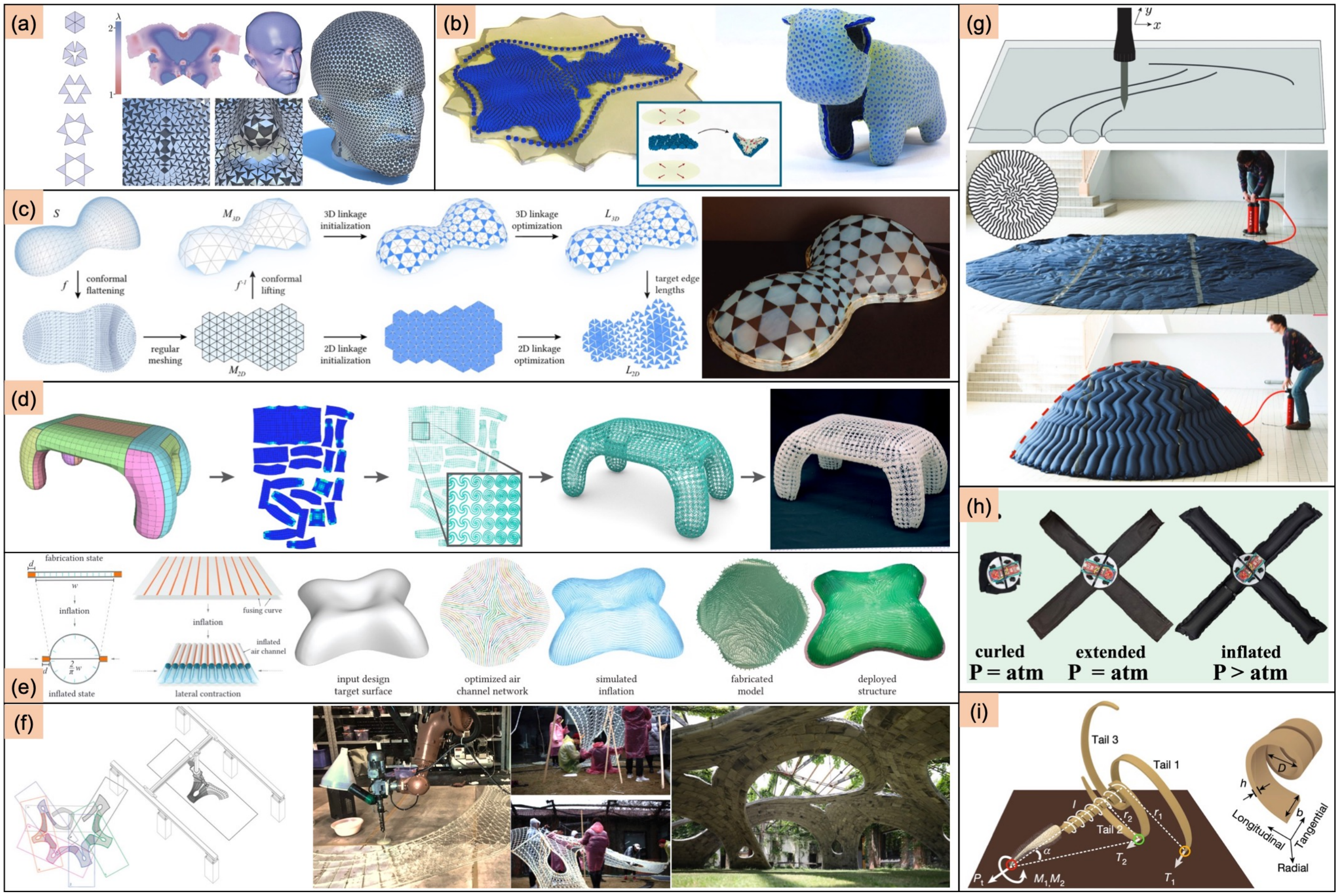


Fig. 11. Expanded studies based on developable surfaces: (a) ”Beyond Developable” based on Auxetic structures approximates various surfaces [107]. (b)”CurveUps” structure with pre-stressed fabrics [102]. (c) Deployable structures of surfaces via Auxetics [108]. (d) ”FlexMaps” as an elastic structural material enhance the fitting of the target shape [126]. (e) A construction method of free-form surface by inflatable and developable structrues [127]. (f) Path printing by robotics for quickly construction of shell [128]. (g) Inflatable and developable structrues used for regular buildings [129]. (h) Integration of developable surfaces with inflatable structures for drones [130]. (i) Self-bending for seeding [131].

segmentation by learning geometric features of the mesh, e.g., “geodesic neural network” [139], “CurvaNet” [140], graph neural network by mesh dual [141], [142], recurrent neural network by random walks on mesh [143], “subdivision-based CNN” [144], and diffusion model based on mesh segmentation [145]. Most AI-based mesh segmentation focuses on semantic recognition, like identifying chair components (back, seat, legs). Although there is no relevant research for developable surfaces, we consider this a promising research direction of shape segmentation for piecewise developable. For example, building datasets by manually categorizing shapes (dividing mesh faces into categories) and training a Graph Convolutional Network (GCN) for predictive classification is a viable approach.

Complexity of Operation. Through interaction, more precise cutting and flattening results can be obtained, and their reliability and stability can be enhanced. However, current shape cutting and flattening operations are relatively complex, and operators must follow the rules with restrictions [41], [59], [64], [126]. Therefore, one approach is similar to semantic-based segmentation [134]–[136], [138] and geometry-based mesh segmentation [139]–[145]. For shape segmentation problems, GCN classification models can be trained to segment meshes into several parts suitable for flattening. Regarding surface flattening, efficient surface unfolding models can be trained by creating a dataset for flattening, speeding up the flattening process.

Digitization to Physicalization. Influenced by real materials and fabrication processes, there exist differences between digital models and physical models [85], [124], [146]. Many current studies utilize finite element analysis to simulate material behavior and reduce errors before producing the physical shape [28], [29], [89], [111], but this approach is time-consuming. A promising solution to address this challenge is to train AI models with physical shape data to rapidly predict physics-based simulation outcomes [147], thereby a precise and fast design-to-fabrication workflow can be implemented.

Future Work. The challenges mostly concern technical and operational aspects. However, a more critical question is to what extent developable surface methods can address specific problems. Therefore, it’s valuable to explore new directions for their application (see Fig. 11).

In the collected studies, certain mechanical materials have been identified to enhance the performance of developable surfaces, thereby improving the developability of target shapes and facilitating 2D fabrication. ”Beyond Developable” [107] introduces auxetic structures with a negative Poisson’s ratio to approximate the target surface, building upon the improved study presented in [108]. In this method, the corners of the various triangles of the structure are linked, while the edges are not, resulting in complex elastic deformation as a whole, with each triangle undergoing rigid rotation. This approach enables local deformability and allows for fitting to various surfaces. ”CurveUps” [102] presents a multi-material structure featuring pre-stressed fabrics. In this method, each mesh face of the target shape is rigidly flattened onto the pre-stressed fabric, similar to kirigami structures approach introduced by Tachi et al. [86] and further developed in [85]. However, kirigami structures utilizes hinge structures to connect adjacent faces, ”CurveUps” utilizes pre-stressed fabric. Upon releasing the tension, all mesh faces can be reassembled into the 3D shape. This approach as a self-deformation structure facilitates the rapid assembly of many segmented components. ”FlexMaps” [126] introduces a complex elastic structural material designed to approximate 3D shapes. In this study, the target surface is flattened using the ARAP method. and then the spiral structures are configured to enhance the fitting of the target shape. It makes a significant contribution to improving the fitting performance of developable surfaces. Inflatable materials [57] usually have a non-negligible membrane strain that allows for non-zero Gaussian curvature. However, the method of piecewise developable surfaces approximating target shapes is still an important method for making inflatable 3D shapes, which based on 2D fabrication is composed of several flat pieces, so some studies employ the method to generate flat pieces.

In recent years, research on ”design to fabrication” has gained increasing attention. Traditional workflows involve independent processes from design to fabrication, which suit collaborative work well. However, integrating manufacturing rules and patterns (such as G-codes) into the design process can implement innovative results that traditional manufacturing cannot achieve. For example, using developable surfaces and robotic technology for path printing allows for the rapid construction of large shells [128]. This research offers insights into combining developable surface fabrication with path printing. Additionally, some studies explore the combination of inflatable and developable structures. For example, optimization of airway paths implements deformation from flat to curved surfaces, enabling the rapid construction of lightweight curved surface architecture [127], [129], transforming developable surfaces into inflatable structures for soft robotics [148]–[150], and utilizing inflatable and developable structures for collision protection in drones [130]. Furthermore, some research focuses on materials with self-bending properties. For example, researchers use self-bending wood to facilitate seed burial underground [131].

The examples above can inspire further research into developable surfaces. Exploring the design of structures (such as combination of path generation and optimization) to enhance material properties for specific requirements (such as self-deformation and increased stiffness), opens up numerous research opportunities and practical applications. Employing relevant geometric design, structural optimization, and tailored fabrication processes can effectively tackle real-world challenges.

8 Conclusion

We summarized complex surface fabrication processes via developable surface approximation from digital modeling to physical modeling, including the sequential strategy and the integrated strategy, as well as main fabrication and assembly methods, we also show the corresponding interactive assistance and design applications. Additional, we summarized a pipeline of the fabrication process from digital modeling to physical modeling. Finally, we discussed current challenges in the study around technical and operational aspects, and suggested opportunities and potential research directions by expanded research.

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